# Preventing Foreclosure in the Telecommunications Local Access Market with Unbundled Local Loops

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## ABSTRACT

In an attempt to provide incentives for carriers to upgrade their networks, the FCC has removed the incumbent carriers obligation to unbundle network elements from fiber optic facilities, including fiber access loops. We show how the effects of vertical integration and tying of services interact to provide a monopoly the ability to completely foreclose a competitor in circumstances such as those faced by competitive telecommunications firms today. We show that unbundling of network access elements is good for social welfare generally while not hurting the firms forced to unbundle their elements. We identify subadditivity of element costs as the primary condition under which unbundling provides a positive benefit. We use these results to support recommendations concerning the regulation of new telecommunications facilities, in particular fiber access facilities. We suggest that a modest period after an upgrade in which the firm owning network access elements does not have to unbundle provides firms with the incentives needed to upgrade their networks, while maintaining the long-term benefits of unbundling with respect to social welfare.


## INTRODUCTION

One of the major conundrums in the current legal situation facing the telecommunications market, and the incumbent local exchange carriers (ILECs) in particular, is the issue of technology and service innovation. The ILECs hold that the obligation to unbundled facilities present in the Telecommunications Act of 1996[1] (the Act) forces them to give away any benefits they might accrue from upgrading their networks, and so they seek relief by restricting such unbundling obligations to existing facilities, not new facilities and services. Fiber optic loop access facilities (known as fiber to the premises or FTTP) are at the heart of this debate.

The Federal Communications Commission's (FCC) Triennial Review Order (TRO)[2] addressed the FTTP issue by ruling that newly constructed fiber facilities did not have to be unbundled by the ILECs[3]. This order gave the ILECs substantial freedom from unbundling requirements with respect to all the fiber they have or will install going forward. The intent is to provide the incumbents a strong incentive to initiate a massive upgrade to their networks. However, these changes fail to mitigate potential abuse by

[1] Telecommunications Act of 1996, Pub. L. No. 104-104, 110 Stat. 56. The 1996 Act amended the Communications Act of 1934, 47 U.S.C. § 151 *et seq.* We refer to these Acts collectively as the "Communications Act" or the "Act."

[2] *Review of the Section 251 Unbundling Obligations of Incumbent Local Exchange Carriers*, CC Dockets 01-338, 96-98, 96-147, Report and Order and Order on Remand and Further Notice Of Proposed Rulemaking. FCC 03-36, released August 21, 2003 (*Triennial Review Order* or TRO).

[3] At this writing, the federal courts have overturned much of the FCC's TRO order, and the FCC is working out their response to this court decision. However, that does not invalidate the analysis here, which looks at the core economics of local access, and unbundling of the access facilities controlled by incumbents (broadly defined) within this market.

carriers in the form of monopoly rents. Under the Act, as implemented by the FCC's TRO, incumbents must unbundle copper access plant, but are under virtually no obligation to unbundle new fiber optic facilities. We show below that this has a high probability of allowing the owner of such fiber facilities to maintain a monopoly, or at least dominant, position in this market. This paper also shows that consumer choice and competitive provider access via unbundled access to this fiber can protect against such abuses, while increasing social welfare.

This paper provides a model of the economics of such facilities, in order to develop an appropriate regulatory approach to such facilities within a competitive telecommunications market, and applies game theory to analyze the model for its impacts on incumbents and competitors. The models focus on the microeconomics of local competition between two or more carriers providing local access services and a bundle of network-based telecommunications services in a single, small service area. While not specifically identified, these telecom services are comparable to unbundled local access facilities, plain old telephone service (POTS) as currently offered, Internet access services, and enhanced services of various kinds.

Based on this analysis, we propose a compromise to the unbundling issue, drawing on the concepts of the protected monopoly period for patented devices, to provide a way to prevent longer-term localized monopolies in fiber access facilities. In the domain of intellectual property, the law has long recognized that a limited period of protected monopoly provides a sufficient and indeed substantial incentive to individuals and companies to innovate. We suggest that regulators borrow the concept of the time-limited monopoly, and apply it to the domain of telecommunications local access facilities. The analysis in this paper supports this proposal.

The paper recommends that these unbundling rules apply to any local exchange carrier (LEC) and not just the incumbents as defined in current law. It argues that an incumbent carrier should be any owner of any shared structures or facility used to provide telecommunications services and that has been in use for a period greater than some predetermined time. Once a shared structure or facility has been in use for this period, it becomes subject to unbundling obligations. Conversely, any carrier can be a new entrant in a local market if its facilities are newer, and is thus not obligated to unbundle. Thus, this paper presents a balanced proposal intended to limit the opportunity for the future monopolization of local access facilities by any carrier while providing an economic incentive to construct new and improved facilities now and in the future.[4]

The plan of the paper is as follows. Section 2 reviews the literature on the economics of local access services. In section 3, we look at incumbents and the potential that they have to foreclose a competitor attempting to provide services in a local

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[4] As a matter of full disclosure, the author is not a lawyer, but rather an engineering/business analyst with significant exposure to the 1996 Act and the FCCs regulations resulting from it. Thus, the focus of the paper is on the economics of this concept, not the legal aspects. The author recognizes that implementing the suggestions in this paper would require significant changes to both federal legislation and the code of federal regulation. Such an effort would require substantial further work in the legal and legislative domains.

telecommunications access market. Section 3 relies on an understanding of how a monopoly carrier will set a service price for a service with strong network effects. Appendix A provides an abridged version of Shy's (2001) monopoly price model under conditions of network effects and his analysis of this process, and we strongly recommend that the reader review this material before proceeding with the remainder of the paper. In section 4, we show that a near monopoly can be effectively constrained by competitors and resellers if regulators require an unbundling mandate. We also address the question of which facilities should be unbundled and why. This includes both technological and economic considerations. Section 5 provides some conclusions.

## LITERATURE REVIEW

Under the Act, as implemented by the FCC's TRO, incumbents must unbundle copper access plant, but are under virtually no obligation to unbundle new fiber optic facilities. We postulate that this is likely to lead to a new monopoly wherever average marginal costs are large relative to the potential revenues the facility can generate, or where a provider can provide, for any of a number of reasons, an exclusive and desirable service that other providers and/or existing facilities cannot provide.

This is, obviously, not a new postulation. However, our approach is different in that our model looks at the microeconomics of a single neighborhood, not at the industry or carrier level. A review of the literature found any number of articles discussing the economics of natural monopolies in the telecommunications industry. However, we found only a few authors that have looked at the economics of the local access plant isolated from the remainder of the network.

Shy (2001) presents a whole volume on the economics of network effects. His analysis and economics are excellent, but he does not apply them to the local access plant specifically. This paper uses many of his concepts and reiterates some of his logic to demonstrate the impacts of network effects on a carrier's competitive strategies. In particular, appendix A of this paper presents an abridged version of Shy's development of a monopoly price for a single provider offering a service with strong network effects. We use the results of this analysis throughout this paper, applying them to the local access market.

Berg and Tschirhart (1995) show that any unique offering under conditions of subadditivity can lead to a monopoly that requires some form of regulatory oversight. Their model demonstrates that for any pair of competing local access service providers, the one with any single unique and desirable service will be able to use that service to cover their base costs. All other costs, even when equitably shared out among services offered by that carrier, will be lower than the marginal costs for the same services offered by a carrier using any technology with lower capabilities. The end result is that the provider of the unique service is able to undercut the prices of all other services by all other providers and gain a natural monopoly, assuming that delivery of the services entails some barrier to entry such as a large sunk cost (say the local access plant). The end result is that the provider with the best delivery mechanism or any unique and desirable service offering will have a sustainable natural monopoly, until such time as another provider can deliver a superior service bundle based on some incremental new offering that cannot be delivered over the existing delivery platform. We demonstrate this same finding using a different model in the next section.

Beard, Ford and Spiwak (2002) formalize and specifically model the underlying economics of telecommunication in the U.S., a model that most people inside the telecommunication industry understand intuitively. In their paper, they develop a theoretical model of the local access market as a two-stage industry with large sunk costs and strong economies of scale. They then use this model to show that in such a market, any vertically integrated carrier has little to no incentive to develop and offer elements of the upstream (network access) stage as a wholesale offering. Further, there is a strong and directly correlated incentive with the firm's market share in the downstream (retail) market. They go on to postulate that no government requirement will successfully overcome these fundamental economic forces. However, using the model, they show that entry by an alternative wholesale only (network access) provider in the upstream stage can be both economically profitable and has the potential to eventually force the incumbent to voluntarily split itself up in order to effectively compete in each market.

Kaserman and Ulrich (2002) use the successive monopoly model of vertical integration to demonstrate how a vertically integrated supplier can use a monopoly in one stage to maintain a monopoly price in the other stage of a two stage vertical market. They go on to demonstrate that resellers can have a powerful market impact, if there is even a modest amount of effective competition in every stage of the market. Further, they showed that as long as the ILECs must resell (or unbundle) every component of their service, it does not matter how fragmented the CLECs are, they can jointly force competitive pricing on the ILEC, if the CLECs achieve a sufficient level of penetration at each stage of the market. Conversely, they demonstrate that if the ILEC maintains an element of monopoly, it can leverage this control into monopoly prices across any service dependant on the monopoly component.

Weisman and Kang (2001) demonstrate that a multi-stage monopoly may produce downstream discrimination when the vertically integrated provider (VIP) is no less efficient than its rivals in the downstream market, but discrimination does not always arise when the VIP is less efficient than its rivals.

Maher (1999) develops a generalized translog cost function of access costs at the local level, and concludes that, contrary to popular belief, cost-based rates at the local level would not be prohibitively high and would not threaten universal service objectives.

Hazlett (2002) provides some indirect support for a protected monopoly period. This support takes the form of a qualitative analysis of the logic of investment and risk transference, including an analysis of the impacts of free riders and zero-cost options generated by a regulatory obligation to unbundle facilities or resell services. This is contrasted with the use of contracts to limit or share such risks before the investment is committed to the installation of a sunk facility.

For a review of other literature concerning attempts to determine if local services provided by the ILECs are a natural monopoly, see Fuss (2002) and Wilson and Zhou

(2001). Wilson and Zhou conclude that economics prove a natural monopoly still exists. Sung & Gort (2000) provide yet another study supporting this thesis. While useful, these studies look at the whole of a provider's operations in a large territory, and are not specific to a particular location.

## Analysis of Foreclosure Opportunities

In this section, we analyze the ability of one competitive local access provider to foreclose a direct competitor in a small local area. We use a game-theoretic approach for this analysis, the economics of network effects, and the concepts of switching costs. We show that foreclosure on the part of an incumbent is possible, presenting the opportunity for one carrier to create a sustainable monopoly. The following section shows that a near monopoly can be effectively constrained by competitors and resellers if regulators require an unbundling mandate. The result improves social welfare generally.

Shy (2001) introduces a methodology that can be used to show if facility owners are able to foreclose providers with lower quality facilities. We use this methodology and notation throughout the remainder of this paper. This methodology includes two orthogonal elements that align the results with known historical examples. The first is the inclusion of a factor that represents the consumers disutility costs when taking service from the less favored provider of a service. This factor can represent the preference of the consumer for the service bundle from a carrier, simple loyalty to a particular carrier, or non-negligible switching costs to change carriers.

The second part of the methodology represents the network effects associated with the utility the service provides to the consumer. This captures the growth in the usefulness of the network to the consumer as the network grows in size. This approach is taken from Shy (2001) and an abridged version of his approach is presented in appendix A. (Readers unfamiliar with this approach are strongly encouraged to review appendix A before proceeding with the remainder of this analysis.) We then look to see how these two effects interact to provide a monopoly the ability to completely foreclose a competitor in some circumstances. Finally, we estimate the social welfare impacts of several alternative approaches to determine which has the best outcome from that perspective, and whether this best outcome can occur without regulatory intervention. We use these results to support the recommendations made below concerning the regulation of new telecommunications facilities, in particular FTTP.

We approach the analysis as follows. In the first step, we introduce the methodology and notation used by showing the game and its results for three independently owned firms, one of which provides basic connection services, and the other two which provide competitive telecom services that are complements to the connection service. In the second step, we see that if the connection provider merges with one of the telecom service providers, foreclosure of the competitor is possible and profitable for the merged firm under certain uncommon conditions. We then improve the assumptions and see that foreclosure is less likely. In the third step, we show that full deregulation leads to competition without the possibility of foreclosure when both the facilities and services are substitutes. Finally, we look at the case where the two facilities provide substantially different service sets, in particular where a newer facility is the only one able to offer a desirable service, in addition to the entirety of the services offered by older facilities. This last case leads to foreclosure of the less capable provider.

THREE PROVIDER TWO SERVICE GAME

Shy (2001, p. 155) discusses the digital convergence issue with a simple model of three providers and two services. In his model, firms A and B provide phone service, and firm C provides Internet access service. Reversing his example but following his logic and notation, we look at the case of one provider of basic connection facilities, and two providers of downstream services. This can be thought of as a fully separated loop (or access) wholesale operator A and two full service telecom service providers denoted as B and C, such as might occur after a regulator structurally separates an incumbent carrier into loop wholesale and service level retail operations. It also mimics the US separation of local and long distance firms after the breakup of AT&T in 1984.

A typical consumer must take one unit of connection service from firm A and one unit of telecom services from either firm B or firm C. For the moment, we assume that production of both the connection and the telecom services is costless. We begin with $2\eta$ consumers, evenly divided in their preferences for the services offered by firms B and C. Let $\beta$ be the gross consumer utility parameter (i.e. the value of the service to the consumer), and $\delta$ be the disutility factor that consumer faces or perceives from obtaining phone service from the less desired firm. As Shy (2001) notes, not all consumers switch to the lowest cost provider. This may occur because they prefer the service bundle from a carrier, are simply loyal to a particular carrier, or face non-negligible switching costs to change carriers. The value of $\delta$ represents this disutility in the following analysis.

We state the preference function for each type of consumer as follows:

$$U_B \overset{def}{=} \begin{cases} \beta - p_A - p_B & \text{buys } A \text{ and } B \\ \beta - \delta - p_A - p_C & \text{buys } A \text{ and } C \\ 0 & \text{Otherwise} \end{cases} \tag{1.}$$

$$U_C \overset{def}{=} \begin{cases} \beta - \delta - p_A - p_B & \text{buys } A \text{ and } B \\ \beta - p_A - p_C & \text{buys } A \text{ and } C \\ 0 & \text{Otherwise} \end{cases}$$

where $p_i$ represents the price paid by the consumer for each of the three services.

As stated, these utility functions also imply that the connection and telecom services are perfect complements. We make this assumption so that if we can show that foreclosure is unlikely when the services are complements, than we also show that foreclosure is even less likely when consumers perceive the two services as separate.

To show that consumers highly value their telecom service provider, we make the initial assumption that $\delta < \beta < 2\delta$. In other words, the services offered by firms B and C are sufficiently different that consumers value the differences.

Remembering that each of the three firms is independently owned, we look for a Nash-Bertrand equilibrium in prices. Observation indicates that there is a unique equilibrium point. This leads to the following proposition.

**Proposition 1**: When there are three independent firms in an industry, the following triplet of prices constitute a Nash-Bertrand equilibrium:

$$p_A = \beta - \delta \qquad p_B = \delta \qquad p_C = \delta \tag{2.}$$

Thus firm A sells a unit of connection services to each consumer, firm B sells one unit of telecom services to each B-oriented consumer, and firm C sells one unit of telecom services to each C-oriented consumer. The firms earn profits of:

$$\pi_A = 2\eta(\beta - \delta) \qquad \pi_B = \eta\delta \qquad \pi_C = \eta\delta \tag{3.}$$

**Proof**. Firm A cannot raise its price, or every consumer would abandon the services offered. The same holds for firms B and C. Further, as firm A sells to every consumer, it cannot expand its market share by lowering it's price. Firm B cannot expand its market share at the expense of firm C without setting it's price to $p_B = \delta - \delta \leq 0$. Thus, it cannot increase its profit by undercutting its rival. The same holds true for firm C.

If we relax the assumption above to the extent that $2\delta < \beta$, firm A gains through an increase in its relative share of the overall market, while firms B and C lose.

Firm A must make a positive return to stay in business, which is necessary for the successful offering of the telecom services by firms B and C. Similarly, without the services of B and/or C, firm A has no reason to offer its connection services. Thus, firm A will not force $\delta$ to zero.

Applying this in practice, the connection service provider will have a relatively high cost structure compared with the disutility value $\delta$, and $\delta$ will be small relative to $\beta$. The monopoly connection firm will be able to extract most of the rents, leaving the telecom service providers to fight for market share and modest profits at best. Historically, the US long distance market in the period from 1985 to 1995 looked exactly like this, with the long distance carriers paying almost half of their revenues to the incumbent LECs in the for of access charges[5].

UNBUNDLED ELEMENT GAME WITHOUT TYING

A second form of the same game allows firm A to merge with firm B and tie the services A and B together. Shy (2001) presents this game with the downstream service as the monopoly with a constant $\beta$, and shows that the competing upstream provider can be foreclosed by the merged firm, but at a loss of profits for the merged firm.

A variation that is more interesting has the monopoly on the upstream side, without the tying. This variation of the analysis conforms to the end state of our main

[5] See the annual reports of AT&T, MCI, or Sprint for this period. All report their access charge expenses in order to clarify their financial reports.

proposal to force facility owners to unbundled access to their facilities after the protected period, as well as the current situation in the provision of local telephone services where the Telecom Act of 1996 requires incumbent LECs to unbundle their local access loops.

The main concern in this analysis is the opportunity that the merged firm AB has to foreclose the independent telecom service provider C. Again using the notation of Shy (2001), we see that foreclosure of firm C is both possible and profitable to the merged firm AB. For clarity, we assume that C-oriented consumers pay two bills, purchasing the connection service A separately from the telecom service C.

**Proposition 2**: Merged firm AB sets a price of $p_A \leq \beta - \delta$ and $p_B = 0$ for their connection and telecom services, respectively, which gives $p_{AB} \leq \beta - \delta + 0$. This overcomes the resistance of C-oriented consumers, and drives firm C out of business, as C must set $p_c = 0$.

**Proof**. Given the bundled offer above by firm AB, the utility of C-oriented consumers is $U_C = (\beta - \delta) - p_A = 0$, thus firm C will not sell, and these consumers will not be served. The profits of firm AB in this foreclosure case are $\pi_{AB} = \eta(\beta - \delta)$, which is lower than in the competitive equilibrium case where:

$$p_A = \delta \qquad p_B = \beta - \delta \qquad p_C = \beta - \delta \tag{4.}$$

$$\pi_{AB} = 2\eta\delta + \eta(\beta - \delta) = \eta(\beta + \delta) \tag{5.}$$

and

$$\pi_C = 2\eta(\beta - \delta). \tag{6.}$$

If merged firm AB lowers the combined price by another $\alpha$, where $\alpha$ is very small, the price becomes $p_{AB} < \beta - \delta - \alpha$, and C-oriented consumers will buy from firm AB. The resulting profits for firm AB are

$$\pi_{AB} = 2\eta(\beta - \delta - \alpha) \tag{7.}$$

which is higher than in the competitive case when $3\delta - 2\alpha < \beta$.

Thus, there is incentive on the part of the merged firm AB to foreclose on firm C when the differences between the telecom services B and C are substantially less than the overall value of the combination of the connection service and either of the two services B or C. This conclusion differs from Shy (2001) in that Shy makes the assumption that $\delta < \beta < 2\delta$. With Shy's assumption, foreclosure is possible but not profitable for the merged firm.

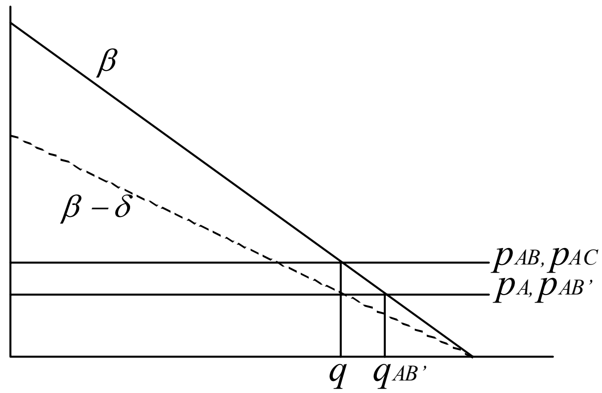
We submit that $3\delta < \beta$ is much more likely than $\delta < \beta < 2\delta$ in the provision of local telecommunications access (connection) services, where there are high fixed and sunk costs, and only a modest level of differentiation between the various basic telecommunications service offerings. Thus, we would conclude that regulatory control is necessary to prevent a vertically integrated firm offering an access service from foreclosing any competing firm dependant on that same access service for the provision of its telecom services. This finding concurs with the analysis by Berg and Tschirhart (1995).

ALTERNATIVE UNBUNDLED ELEMENT ANALYSIS

We next define a linear $\beta$ function, to more accurately reflect the variable value consumers place on particular services. We note that a true function of $\beta$ is unlikely to be either constant (as above) or linear. Following Shy's definitions (2001) with modifications, we begin by considering a continuum of $2\eta$ potential consumers uniformly indexed by $x$ on the unit interval [0, 1] with density $\eta > 0$. We assume for simplicity that exactly half of the consumers are oriented towards the downstream telecom service offering B from firm AB, and the other half are oriented towards the offering from firm C, and an equal distribution of each type across the continuum. Both sets of consumers must also subscribe to service A from firm AB in order to gain access to their preferred downstream service. We interpret consumers indexed by a low $x$ as those with a high willingness to pay ($\beta$), and consumers indexed with a high $x$ as those with a low willingness to pay. This provides a (linear) *cumulative density function*, which tells us for any value of $x$ how many consumers there are with index types between zero and $x$, and shows that consumers indexed by $x > \hat{x}$ will not subscribe to the service. Thus for example there are $\eta$ (half the population) who are types indexed on [0, ½].

We denote with $q$, $0 \le q \le 1$, the total number of customers who subscribe to a telecom service and with $p$ the price charged to subscribe to the service. Firm AB, having an unregulated monopoly in the connection service A, will set $p_{AB}$ to the monopoly level, which is the price $p_m$ that maximizes their profits. Shy (2001, p. 110ff) derived this monopoly level in the case of a single service monopoly with network effects, and shows that the firm will set a price such that $q = 2/3$ to maximize their profits from the monopoly service A combined with the in-house service B. We will call this single-firm optimal price the monopoly price $p_m$ in this and all subsequent analysis. See appendix A for an abbreviated version of Shy's derivation of this result. We also assume for the moment that $3\delta = \beta$ for all $\beta$. This is shown in figure 1.

**Figure 1.**




**Proposition 3**: There exists a Nash-Bertrand equilibrium, where $q$ is the profit maximizing level for firm AB for the combination of connection service A and telecom service B, $p_{AB} = \beta_q$, $p_A = \beta_q - \delta_q$, and $p_B = p_C = \delta_q$. This equilibrium is unique.

**Proof**. Taking $q$ as the profit maximizing point for firm AB for the moment (see appendix A for this derivation), we look to see if a change in prices is profitable for either firm. We divide the problem into several parts, looking at each possible price change independently.

Suppose that firm AB, in an effort to undercut Firm C, set $p_B = 0$. This lowers the total price to $p_{AB'}$, and shifts $q$ to $q_{AB'}$. This reduces firm AB's total income from B-oriented consumers, while income from C-oriented consumers depends on the reaction of firm C. Because the utility function of C-oriented consumers is still:

$$U_C \overset{def}{=} \begin{cases} \beta - \delta - p_A - p_B & \text{buys } A \text{ and } B \\ \beta - p_A - p_C & \text{buys } A \text{ and } C \\ 0 & \text{Otherwise} \end{cases} \tag{8.}$$

the utility of these consumers is $U_C = \beta - p_A - p_C = \beta - \delta - p_A - 0$, and they will not shift to service B. Therefore, firm C does not need to respond to this price change on the part of firm AB. Increasing $p_B$ naturally does not work, as its only effect is to shrink the number of consumers that will subscribe to service B, which lowers profits for firm AB.

We note that changes in $p_A$ have no impact on firm C's ability to do business, though it will change the number of consumers with $\beta > p_A + p_C$, thus shifting $q$ to the right, and shrinking $\delta$ if $\delta$ is a function of $\beta$ as assumed here. This is easy to see with our earlier assumption that C-oriented consumers pay two bills, purchasing the connection service A separately from the telecom service C. The unique equilibrium:

$$p_{AB} = \beta_q,\ p_A = \beta_q - \delta_q, \text{ and } p_B = p_C = \delta_q, \tag{9.}$$

where $q$ is the profit maximizing point for firm AB, results.

We conclude that firm AB cannot foreclose or undercut firm C in this case without tying, and that firm AB will live with the competition from firm C. This reverses the finding immediately above, with it's admittedly extreme assumption of a constant $\beta$. This implies that regulatory control is not necessary to prevent a vertically integrated firm offering an access service from foreclosing any competing firm dependant on that same access service for the provision of its telecom services, in the presence of significant product differentiation. We also observe that every consumer that takes one of these offered services does so from their preferred firm, a point that we will use in our analysis of social welfare below.

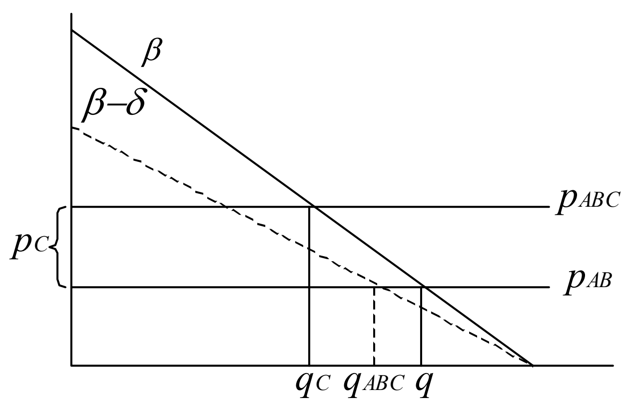
UNBUNDLED ELEMENT ANALYSIS WITH TYING

Looking next at the case where firm AB is able to tie its two services together, Shy (2001) shows that foreclosure is possible with a constant $\beta$. We conclude the opposite, when $\beta$ is not a constant.

**Proposition 4**: With a linear $\beta$, foreclosure of firm C is not possible when firm AB ties services A and B together.

**Proof**. Firm AB sets an initial price $p_{AB} = p_m$, leading firm C to set a price $p_C > 0$ such that the total cost for a C-oriented consumer is $p_{ABC} = p_{AB} + p_C$. This price $p_C$ will be the monopoly price as determined by firm C, over the range of potential C-oriented consumers $0 < \hat{q} < q$ and will determine the location of $q_{ABC}$ relative to $q_C$. Figure 2 illustrates this.

**Figure 2.**




Lowering the price $p_{AB}$ has a positive effect on firm C, giving it room to serve additional consumers and increasing its profits, while firm AB lowers its profits. Similarly, if firm AB raises their price, then the C's consumer base shrinks and lowers C's profits. The cost to firm AB is a loss of their own consumers with a corresponding reduction in profitability, remembering our assumption that $q$ corresponds to the profit maximizing point for firm AB. Thus, foreclosure is not possible. However, if $p_C > \delta_{q_C}$ there is an impact on social welfare, as the number of consumers that take service from their second choice firm is not zero (the consumers that lie in the range of $q_C < \hat{q} < q_{ABC}$)

and consumers taking service from firm C must pay twice for a service that they only use once. We state this as

$$\Delta W = -\sum_{x=q_C}^{q_{ABC}} \delta_x \,. \qquad (10.)$$

As we see later, prices do not affect the level of overall social welfare, so no price term is included here. We provide an enhanced analysis of this impact in the section on social welfare below.

DUOPOLY GAME

In many locations, two existing facilities are capable of supporting most basic telecom services, the LECs copper or fiber, and the cable television system's hybrid-fiber-coaxial (or HFC) lines.

Continuing our analysis of the cases that currently occur in the telecommunications industry, we look at a duopoly of providers AB and AC, both offering a bundle of connection (or access) services and a set of basic telecom services that ride over the connection service. We assume that the connection services are perfect substitutes, on parallel facilities, with firm C's version denoted as $A'$. We further assume that the telecom services are substitutes differentiated by the disutility value $\delta$, and that $\delta$ is small relative to $\beta$, such that $3\delta < \beta$, and where $\beta$ is for the moment a constant. Shy (2001) calls this case the complete deregulation case, but approaches it as a single stage game, with $\delta < \beta < 2\delta$, whereas this approach is effectively a multistage game. The outcome is the same with either approach.

We restate the consumer utility functions as:

$$U_{AB} \stackrel{def}{=} \begin{cases} \beta - p_{AB} & \text{buys } AB \\ \beta - \delta - p_{A'C} & \text{buys } A'C \\ 0 & \text{Otherwise} \end{cases} \qquad (11.)$$

$$U_{AC} \stackrel{def}{=} \begin{cases} \beta - \delta - p_{AB} & \text{buys } AB \\ \beta - p_{A'C} & \text{buys } A'C \\ 0 & \text{Otherwise} \end{cases}$$

**Proposition 5**: The equilibrium prices are $p_{AB} = \beta$ and $p_{A'C} = \beta$. Undercutting is not profitable, and foreclosure is not possible.

**Proof**. If firm AB attempts to undercut firm C, the utility function implies that it must set $p_{AB} = \beta - \delta - \alpha$, where $\alpha$ is very small, to attract any of the C-oriented consumers. Firm C reacts, naturally, but only enough to keep its consumers. Firm C will lower it's price by $\alpha$ to keep the difference in prices less than or equal to $\delta$, thus keeping C-oriented consumers from switching. This continues until $p_{AB} = 0$, at which point firm C will have a price of $p_{A'C} = \delta$ and a positive profit value of $\pi_{A'C} = \eta\delta$. Thus undercutting and foreclosure are not possible, and both firms will maintain prices $p_{AB} = \beta$ and $p_{A'C} = \beta$.

If we use a linear $\beta$ instead, we find again that undercutting and foreclosure are not possible. The logic follows thus. Firm AB determines the ideal monopoly price $p_m$ for AB-oriented consumers and sets $p_{AB} = p_m$. Firm C does the same, setting $p_{A'C} = p_m$. In an attempt to foreclose firm C, firm AB must lower its price to $p_{AB} = p_m - \delta - \alpha$. As above, firm C merely needs to respond with a price of $p_{A'C} = p_m - \alpha$, maintaining a price differential of $\delta$ or less to keep its consumers. Firm AB cannot thus undercut firm C, and both firms will chose a price that maximizes their profitability.

Note that if the two firms set prices such that $|p_{AB} - p_{A'C}| > \delta$, the firm with the higher price must reduce its price to at least within $\pm\delta$ of the lower firm to maintain its customer base. This shows that where it is economically cost effective to install and maintain multiple connection facilities offering largely similar service bundles, a duopoly will develop, and the price for the respective services bundles will be similar and set at a level that maximizes the profits of the more cost efficient producer. This occurs without any intervention on the part of the regulator. The network effects discussed in appendix A in turn determined the price.

We look to the impact of this analysis on social welfare. Because consumers are free to choose their provider and will do so, the total value of the $\delta$ terms in the consumer utility function for all consumers, taken by consumer, is zero. We will use this information below.

<u>Differential Service Game with Tying</u>

In this game, we look at the case of a duopoly where one of the two competitors has the ability to deliver an extra, desirable service D. This is the current situation with CATV providers over LECs, where the CATV industry can deliver voice, video and data streams over their HFC facilities, while the LECs are limited to voice, data, and a very limited video capability, at best. Thus, we can think of D as video service today. Of interest to the analysis at hand, this may also apply to future FTTP installations, where the fiber is able to offer some new and desirable service that cannot be offered using today's HFC facilities. Other factors that might limit the availability of D to one provider include exclusive contracts, patent or copyright protection, or other non-technical factors.

We next identify the consumer utility function for our two consumer types, with type AB oriented to the services offered by firm A, and type AC oriented to the services offered by firm C, consistent with our previous usage of these definitions:

$$U_{AB} \stackrel{def}{=} \begin{cases} \beta + \kappa - p_A - p_B - p_D & \text{buys A, B, \& D} \\ \beta - p_A - p_B & \text{buys A \& B only} \\ \beta + \kappa - \delta - p_A - p_D - p_{A'C} & \text{buys A, C, \& D, from 2 firms} \\ \beta - \delta - p_{A'C} & \text{buys A \& C only} \\ 0 & \text{Otherwise} \end{cases} \tag{12.}$$

$$U_{AC} \stackrel{def}{=} \begin{cases} \beta + \kappa - \delta - p_A - p_B - p_D & \text{buys A, B, \& D} \\ \beta - \delta - p_A - p_B & \text{buys A \& B only} \\ \beta + \kappa - p_A - p_D - p_{A'C} & \text{buys A, C, \& D, from 2 firms} \\ \beta - p_{A'C} & \text{buys A \& C only} \\ 0 & \text{Otherwise} \end{cases}$$

where $\kappa$ is the positive extra consumer benefit of service D above $\beta$. We assume that $\kappa$ is proportional to $\beta$ for all $\hat{q}$, and that every consumer will buy service D if the price is below their price threshold. Service D is only available from firm A. Both firms A and C provide the supporting connection service *A*, with firm C's version denoted as $A'$. Not all of these options will be available to the consumer in the alternatives below, since the various tying options will eliminate some of the possibilities shown for some alternatives. Others are just very unlikely.

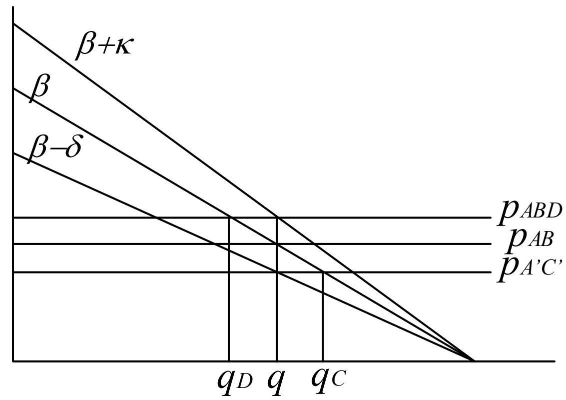
To simplify, we assume that tying is occurring. If tying is not required on the part of firm A, then the three services are independent, and the previous analysis of the duopoly applies. This leaves three possible combinations of tying: *A*, *B*, and *D*, *A* with B only, and *A* with *D* only. Since we want to know what will happen when firm A offers *D* to existing users of both *B* and *C*, we need to look at the interaction of the $ABD$ combination with $A'C$, and the interaction of the pair of bundles *AB* plus *AD* with $A'C$.

Note also that the value $\delta$ is not associated with the service *D*. Since it represents the disutility of taking a service from a less desirable source, and there is only one source for *D*, we assume that there is no $\delta$ associated with taking the combination of *A* and *D* from firm A by a C-oriented consumer. However, as shown, such a combination may incur the cost of purchasing the transport service twice.

If $\beta$ is a constant, then firm A can foreclose firm C by setting $p_{AB} = 0$ and $p_D \leq \beta$. This is the same as Shy's (2001) proposition 6.11, where firm C will leave the market as it cannot compete with the offering price of zero for service *AB* from firm A for its one competitive service bundle, $A'C$. As before, the more realistic case uses a linear function for $\beta$, and provides the opposite outcome.

**Proposition 6**: If $\beta$ and $\kappa$ are linear, there is a Nash-Bertrand equilibrium wherein firm A cannot foreclose firm C, but firm C will retain only those consumers indexed by $\hat{q}$ in the interval $q_D < \hat{q} < q_C$, and will lose all customers indexed by $\hat{q}$ in the interval $0 < \hat{q} < q_D$. Firm A will set $p_{ABD} = p_m = \beta_q + \kappa_q$, and firm C will lower $p_{A'C}$ to the profit maximizing level $p_{A'C'} = p_{m'}$ that corresponds to the smaller consumer base available to it after the loss of consumers seeking service *D* from firm A. Figure 3 illustrates this.

**Figure 3.**




**Proof**. Begin by looking at the $AB$ vs. $A'C$ market. This is the simple duopoly case discussed above. Undercutting is not profitable, and both firms will set their prices to the profit maximizing level $p_m$. Thus,

$$\begin{aligned} p_m = p_{AB} = \beta_q \qquad & p_m = p_{A'C} = \beta_q \\ U_{AB} = \beta - p_{AB} \qquad & U_{AC} = \beta - p_{A'C} \\ \pi_{AB} = \eta q(p_{AB}) \qquad & \pi_{AC} = \eta q(p_{A'C}) \end{aligned} \tag{13.}$$

as before. Next, firm A offers service *D* under a contract such that the consumer must take all three services as a bundle. As firm A has a monopoly on service *D*, it will set $p_D = \kappa_q$, and maintain $p_{AB} = \beta_q$, as shown in figure 3. C-oriented consumers will take the $ABD$ bundle when $\beta + \kappa - \delta > p_{ABD}$. Thus, C-oriented consumers indexed in the interval $0 < \hat{q} < q_D$ will switch to take the full bundle $ABD$ from firm A.

For illustrative purposes only, figure 3 assumes that $\kappa = \delta$. If $\kappa$ is much larger than $\delta$, then the attractiveness of service D will offset the disutility associated with service B, and more C-oriented consumers will take the full service bundle from firm AB.

Because firm C cannot offer service *D* (which is offered exclusively by firm A), it must live with the restricted (and less attractive) customer base that results. As it has a reduced consumer base, it will have a new profit maximizing level $p_{A'C'} = p_{m'} < p_{A'C}$ at point $q_C$, allowing it to serve consumers indexed in the interval $q_D < \hat{q} < q_C$. The following profit levels represent a Nash-Bertrand equilibrium:

$$\begin{aligned} \pi_A &= \eta(p_{ABD})(q + q_D) \\ \pi_C &= \eta(p_{A'C'})(q_{A'C'} - q_D) \end{aligned} \tag{14.}$$

With this model, firm A can squeeze (but cannot foreclose) firm C by reducing the level of $p_D$ (at the cost of lower profits) by decreasing $\delta$, or by increasing $\kappa$, any of which will attract more consumers to shift to their service. Conversely, firm C can regain consumers by increasing $\delta$, which implies increasing brand loyalty, bundle $A'C$ attractiveness, or switching costs in its favor, all without changing the price of its bundle.

In practice, firm A is likely able to squeeze firm C if service *D* is attractive and $\delta$ is relatively small. Foreclosure will happen if firm C's costs are greater than $\delta$. Conversely, firm C will likely have the advantage of a mature connection facility with a lower operating marginal cost level, brand loyalty, and consumer inertia, allowing it to endure such a squeeze for a considerable length of time.

From a regulator's perspective, $\kappa$ must offset the non-zero values of $\delta$ associated with C-oriented consumers before overall social welfare can rise. Further, since service D is a monopoly, any disutility directly associated with it remains and has an impact on overall social welfare. We discuss this further below.

DIFFERENTIAL SERVICE GAME WITHOUT TYING

Finally, we come to the case where firm A does not tie the new service *D* to the old service *B*, allowing consumers to take bundle $AD$ without service *B*. Begin by noting that C-oriented consumers that take service *D* from firm A have a choice: they can take the whole bundle from firm A, or they can pay for a multiple connection services, *A* and $A'$, and take one service from each provider. Since each provider will be looking to maximize its profits based on this decision, we look for the critical point for these C-oriented consumers. We set the two options equal:

$$\beta + \kappa - \delta - p_A - p_D - p_B = \beta + \kappa - p_A - p_D - p_{A'C} \qquad (15.)$$

and solve for the price $p_{A'C}$:

$$p_{A'C} = p_B + \delta \, . \qquad (16.)$$

Thus, if $p_{A'C} \le p_B + \delta$ then the C-oriented consumer will buy $A'C$ from firm C and $AD$ from firm A. Conversely, if $p_{A'C} > p_B + \delta$ then they will buy $ABD$ from firm A.

We see immediately that $\beta$, $\kappa$, and $p_D$ drop out of the equation. Because $p_A + p_B = \beta_q$, we will need to look at the effects of price shifting from service *B* to service *A* to determine if such shifting can provide an extra advantage to firm A.

**Proposition 7**: Firm A maximizes its profits by setting its prices to

$$p_A = \beta_q \qquad p_B = 0 \qquad p_D = \kappa_q \, . \qquad (17.)$$

Firm C will maintain customers and maximize its profits by setting $p_{A'C} = p_B + \delta$. This is a Nash-Bertrand equilibrium.

**Proof**. Firm A will maximize its profits by setting its prices at the profit maximizing level, which is found at $\beta_q$ and $\kappa_q$, and the sum of its prices will equal $p_{ABD} = \beta_q + \kappa_q$. This is due to the monopoly profit maximization process in appendix A. Because any consumer taking service *D* will also need to take service *A*, including C-oriented consumers, firm A sells $2q$ units of services *A* and *D*. However, if the offering price from firm C for service bundle $A'C$ is low enough, C-oriented consumers will fail to take competitive service *B*. Firm Cs best price is $p_{A'C} = p_B + \delta$ from equation 16 above, as any higher price will cause C-oriented consumers to switch to competing service *B*.

As the prices for services *A* and *B* together must equal $\beta_q$ to maximize profits, and firm A sells twice as many units of *A* as it does of *B*, it will skew its price such that *A* is higher, approaching $\beta_q$. The price of *B* will fall to zero. With $p_A$ at $\beta_q$, increasing $p_B$ will lower profits by raising the total price of service above $\beta_q$. This, due to the monopoly profit maximization process, will result in fewer customers and a lower overall profit for the firm.

We can also show that firm C cannot win any A-oriented consumers to its services. The only exception is when $\delta = 0$ and $p_{A'C'} < p_B$, which is not realistic.

The implications for firm A are that it can give away competitive services for free without hurting profits, provided the costs can be bundled with a monopoly product or service. Foreclosure is not possible theoretically (assuming that $\beta$ is not a constant), but is clearly possible in practice because if firm C's costs are greater than $\delta$ then it cannot stay in business, and firm A will foreclose firm C by the simple act of cost shifting.

Conversely, firm C is severely constrained, living entirely within the level of $\delta$. Thus firm C has a strong incentive to increase its product differentiation. It can do this by developing a strong brand loyalty, widen service specific differences, or increase switching costs to the consumer. In the ideal case, it too will find a service that it can exclusively offer, attracting consumers oriented towards other firms.

The impact on social welfare is straightforward. Because any consumer that takes service from an alternative provider has a positive value for $\delta$, social welfare decrease by the sum of the $\delta$'s for all C-oriented consumers that purchase the alternative services. We state this as

$$\Delta W = -\sum_{x=0}^{q_D} \delta_x \,. \qquad (18.)$$

Again, since service D is a monopoly, any disutility directly associated with it remains and has an additional impact on overall social welfare. This includes the cost of purchased capacity in service B that goes unused by C-oriented consumers.

IMPLICATIONS OF FORECLOSURE ANALYSIS

In the telecommunications local access market, with its high fixed and sunk costs and (at the moment anyway) relatively low product differentiation, we submit that foreclosure is likely for some service combinations without regulatory intervention. However, mandatory unbundling with some form of imputation rule mitigates this, as it separates the monopoly service from the competitive services, and places limits on the access facilities owner's ability to foreclose competitors.

## IMPACT OF UNBUNDLING ON SOCIAL WELFARE

Moving now to the second key element of our analysis, we look at the power of unbundled elements and resale obligations to expand consumer and social welfare without harming the incumbent firms' ability to make a profit. In this section, we characterize the overall social welfare equation for each of the unbundled element games. This produces a large and tedious result that demonstrates that unbundling improves social welfare under one condition. We begin with the general function for social welfare *W*, which is

$$W = \pi' + U' \tag{19.}$$

where $\pi'$ is the total profit captured by all the firms in the market, and $U'$ represents the totality of the consumer's aggregated utility functions. More specifically, we can expand this to cover multiple firms and services with

$$W = \sum_{i=1}^{F} \pi_i + \sum_{k=1}^{\eta'} U_k \tag{20.}$$

where *F* is the number of firms, and $\eta'$ is the total number of potential consumers in the market. We expand each of these components in turn below, with the aim of combining and simplifying the resulting equation so that we can find the maximum with respect to *W* and determine if this is better, worse, or the same as the results for a monopoly provider.

As a matter of representation, we make the following definitions, which follow the nomenclature used by Shy (2001). We define $\eta'$ as a continuum of *potential* consumers indexed by *x* on the unit interval [0,1]. We interpret consumers indexed by a low $x$ as those with a high willingness to pay, and consumers indexed with a high $x$ as those with a low willingness to pay. This provides a *cumulative density function*, which tells us for any value of $x$ how many consumers there are with index types between zero and $x$. Thus for example there are $\eta/2$ (half the population) who are types indexed on [0, ½].

We define $q'$ as the total number of *actual* consumers that take services from all carriers. We can relate $q'$ to $\eta'$ with $\hat{x}$, which is the index of the consumer indifferent to subscribing at a given connection price *p*. Thus, we note that $q' = \eta'\hat{x}$ represents the actual number of consumers taking services in the market.

### CONSUMER UTILITY

We set the consumer utility function as follows, noting that service 1 (unbundled access facilities) are essential complementary inputs to service 2 (basic retail connection services) and service 3 (optional enhanced retail services).

$$U_x \stackrel{def}{=} \begin{cases} \beta - p_1 - p_2 & \text{buys basic only from prefered carrier} \\ \beta + \kappa - p_1 - p_2 - p_3 & \text{buys basic plus enhanced from prefered carrier} \\ \beta - \delta - p_1 - p_2 & \text{buys basic only from alternative carrier} \\ \beta + \kappa - \delta - p_1 - p_2 - p_3 & \text{buys basic plus enhanced from alternative carrier} \\ 0 & \text{Otherwise} \end{cases} \tag{21.}$$

We restate this as a function of the number of consumers of each type. Divide the actual consumers $q'$ evenly into 4 groups as shown in table 1. Table 1 represents consumers' preferences, and does not indicate which provider actually provides the service.

**Table 1: Consumer Counts**

| Preferred Provider: | 1 | 2 |
|---|---|---|
| 1. Require Loop Service | $q_{1,1} = q_{1,2} + q_{1,3}$ | $q_{2,1} = q_{2,2} + q_{2,3}$ |
| 2. Take Basic Retail Connection Service Only | $q_{1,2} = \frac{q'}{4}$ | $q_{2,2} = \frac{q'}{4}$ |
| 3. Take Enhanced Retail Plus Basic Service | $q_{1,3} = \frac{q'}{4}$ | $q_{2,3} = \frac{q'}{4}$ |

We can capture other relationships between these variables as necessary. Additionally, if we wish to study a monopoly or ignore the enhanced service, an appropriate adjustment can be made here, and the relevant terms will vanish from the more general analysis that follows.

Next, we rewrite the overall customer utility function $U'$ as a weighted sum of the number of consumers and their components as follows:

$$\begin{aligned} U' = \sum_{x=0}^{\hat{x}} \beta(x, \eta, \hat{x}) + \sum_{x=0}^{\hat{x}} \kappa(x, \eta_\kappa, \hat{x}) - \sum_{x=0}^{\hat{x}} \delta(x, \hat{x}) - p_{1,1}q_{1,1} - p_{2,1}q_{2,1} \\ - p_{1,2}q_{1,2} - p_{2,2}q_{2,2} - p_{1,3}q_{1,3} - p_{2,3}q_{2,3} \end{aligned} \tag{22.}$$

where $p_{i,j}$ represents the price for each provider/service pair, and $\delta(x, \hat{x})$ represents the disutility function associated with taking service from an alternative to the preferred provider. The disutility factor $\delta(x, \hat{x})$ will be different for each game studied, since it depends on the number of consumers that end up taking a service from their second choice provider for any reason. We cover this in more detail below. Prices fall out of the social welfare equation, as will be seen below, and thus are not explored in detail.

The values for $\beta(x, \eta, \hat{x})$ and $\kappa(x, \eta_\kappa, \hat{x})$ are also interesting. Shy (2001) discusses the economics of network effects in two chapters, both of which use parallel concepts of $\beta$, though only the second uses the more general notation for $\beta$. Shy's section 5.2 on monopoly telecommunication service providers uses a value for the customer utility function of:

$$U_x = \begin{cases} (1-x)q^e - p & \text{if she subscribes} \\ 0 & \text{if she does not subscribe} \end{cases} \quad (23.)$$

In Shy's section 6.3 on digital convergence, the customer utility function is stated as

$$U_A \stackrel{def}{=} \begin{cases} \beta - p_A - p_C & \text{buys } A \text{ and } C \\ \beta - \delta - p_B - p_C & \text{buys } B \text{ and } C \\ 0 & \text{Otherwise} \end{cases} \quad (24.)$$

$$U_B \stackrel{def}{=} \begin{cases} \beta - \delta - p_A - p_C & \text{buys } A \text{ and } C \\ \beta - p_B - p_C & \text{buys } C \text{ and } C \\ 0 & \text{Otherwise} \end{cases}$$

which is clearly a parallel form after accounting for the multiplicity of services. Thus, we propose the following definition of $\beta$:

$$\beta(x,\eta,\hat{x}) = (1-x)q^e = (1-x)\eta\hat{x} \quad (25.)$$

where $q^e$ is the expected number of consumers, $q'$, which is in turn based on the coordination assumption of Shy (2001, page 20)and leads to $q^e = q' = \eta\hat{x}$.

Since we define $\kappa(x,\eta_\kappa,\hat{x})$ to be proportional to $\beta(x,\eta,\hat{x})$, the logic is the same, leading to $\kappa(x,\eta_\kappa,\hat{x}) = (1-x)\eta_\kappa\hat{x}$. Thus our expanded definition of social welfare $W$ is

$$W = \pi' + \sum_{x=0}^{\hat{x}}((1-x)\eta\hat{x}) + \sum_{x=0}^{\hat{x}}((1-x)\eta_\kappa\hat{x}) - \sum_{x=0}^{\hat{x}}\delta(x,\hat{x}) \\ - p_{1,1}q_{1,1} - p_{2,1}q_{2,1} - p_{1,2}q_{1,2} - p_{2,2}q_{2,2} - p_{1,3}q_{1,3} - p_{2,3}q_{2,3} \quad (26.)$$

PROFITS

Next, we introduce the general profit function $\pi_i$ for carrier *i*, which is

$$\pi_i = \sum_{j=1}^{S}\left[q_{i.j}p_{i,j}\right]. \quad (27.)$$

If we include fixed and variable costs, the function becomes

$$\pi_i = \sum_{j=1}^{S}\left[(p_{i,j} - \mu_{i,j})q_{i,j} - \varphi_{i,j}\right], \quad (28.)$$

where $S$ is the number of services the firm sells, $\mu$ is the variable costs per subscriber and $\varphi$ is the fixed costs allocated to the service. We expand on each of these elements next, beginning with costs.

COSTS

We begin by defining the *Imputed Cost Rule*, which requires that a firm's wholesale price $p_{1,1^*}$ for a service be greater than their cost for that service, and less than their retail price. Formally, this is

$$\mu_{1,1} + \frac{\varphi_{1,1}}{q_{1,1}} < p_{1,1^*} \leq p_{1,1} \tag{29.}$$

The alternative, where $p_{1,1} < p_{1,1^*}$ is economically acceptable, but is likely to draw the attentions of law enforcement on the grounds of anti-competitive behavior.

Costs are the tricky bit when unbundling is included, since a large portion of firm 2's cost $\mu_{2,2}$ for their connection service is payment of a wholesale price $p_{1,1^*}$ for the unbundled element. From the customer's perspective, the wholesale price is a moot point, since it only influences the division of their costs between the two providers, and not their overall cost. However, if the wholesale price ( $p_{1,1^*}$ in this case) is too high, the second firm will not be able to cover their additional costs, and will exit the market.

Naturally, a firm with a monopoly on service 1 will seek to maximize its returns on this service, by raising the price as much as possible, even in the wholesale case. The analysis above in the three provider two service game suggests that the optimal price for such a service with network effects is

$$p_{1,1^*} = p_{1,1} = \beta_{\hat{x}} - \delta_{\hat{x}} = (1-x)q_{1,1} - \delta_{\hat{x}} \tag{30.}$$

We assume here that $p_{1,1^*} = p_{1,1}$ for two reasons. First, our topic of interest is local telecommunications access services, which includes access facilities as service 1 that consumers do not purchased directly. Thus, we assume that access facilities have only one price, the wholesale price. Second, this is the worst-case scenario with respect to our analysis, and as such, any changes will benefit the consumer. This assumption also allows us to capture the detail that in reality, a wholesale price will include not just the variable cost, but a share of the fixed costs associated with the service.

Turning first to firm 2, the purchaser of the unbundled elements, we note that their customers will see only one price on a bill for the combination of services that we show as $p_{2,1} + p_{2,2}$. Because firm 2 does not have to charge the same price for the unbundled element, we use the value $p_{2,1}$ for this element of the service, and then make the assumption that $\mu_{2,1} = p_{1,1^*} \leq p_{2,1}$, which has the effect of passing the cost on to the consumer with the difference $p_{2,1} - p_{1,1^*}$ being the markup retained by firm 2. We also assume for simplicity's sake that firm 2 has no additional marginal costs associated with the purchase of the unbundled elements from firm 1. We state firm 2's general profit function as

$$\pi_2 = (p_{2,1} - \mu_{2,1})q_{2,1} - \varphi_{2,1} + (p_{2,2} - \mu_{2,2})q_{2,2} - \varphi_{2,2} + (p_{2,3} - \mu_{2,3})q_{2,3} - \varphi_{2,3} \tag{31.}$$

The profit function for firm 1 is more complex. Taken in parts, we begin with their wholesale profits, stated as

$$\pi_{1^*} = (p_{1,1^*} - \mu_{1,1})(q_{2,2} + q_{2,3}) \tag{32.}$$

An allocation of fixed costs should be properly included here. Since we will merge them with the firms' retail revenues in the next step, we have simplified this equation by ignoring them here.

Continuing with firm 1, we define their retail profit function, in parallel with that for firm 2, as follows:

$$\pi_{1'} = (p_{1,1} - \mu_{1,1})q_{1,1} - \varphi_{1,1} + (p_{1,2} - \mu_{1,2})q_{1,2} - \varphi_{1,2} + (p_{1,3} - \mu_{1,3})q_{1,3} - \varphi_{1,3} \qquad (33.)$$

We combine this with the wholesale profits from equation 32 to give the firms overall profit equation,

$$\begin{aligned}\pi_1 = (p_{1,1} - \mu_{1,1})q_{1,1} - \varphi_{1,1} + (p_{1,2} - \mu_{1,2})q_{1,2} - \varphi_{1,2} + (p_{1,3} - \mu_{1,3})q_{1,3} - \varphi_{1,3} \\ + (p_{1,1^*} - \mu_{1,1})(q_{2,2} + q_{2,3})\end{aligned} \qquad (34.)$$

Combining equations 31 with 34, and substituting the resulting value of $\pi'$ into equation 26 above provides the following, rather lengthy, equation for social welfare:

$$\begin{aligned}W = (p_{1,1} - \mu_{1,1})q_{1,1} - \varphi_{1,1} + (p_{1,2} - \mu_{1,2})q_{1,2} - \varphi_{1,2} + (p_{1,3} - \mu_{1,3})q_{1,3} - \varphi_{1,3} \\ + (p_{2,1} - \mu_{2,1})q_{2,1} - \varphi_{2,1} + (p_{2,2} - \mu_{2,2})q_{2,2} - \varphi_{2,2} + (p_{2,3} - \mu_{2,3})q_{2,3} - \varphi_{2,3} \\ + (p_{1,1^*} - \mu_{1,1})(q_{2,2} + q_{2,3}) + \sum_{x=0}^{\hat{x}}((1-x)\eta\hat{x}) + \sum_{x=0}^{\hat{x}}((1-x)\eta_\kappa\hat{x}) - \sum_{x=0}^{\hat{x}}\delta(x,\hat{x}) \\ - p_{1,1}q_{1,1} - p_{2,1}q_{2,1} - p_{1,2}q_{1,2} - p_{2,2}q_{2,2} - p_{1,3}q_{1,3} - p_{2,3}q_{2,3}\end{aligned} \qquad (35.)$$

Canceling out the price terms can reduce this to

$$\begin{aligned}W = (p_{1,1^*} - \mu_{1,1})(q_{2,2} + q_{2,3}) + \sum_{x=0}^{\hat{x}}((1-x)\eta\hat{x}) + \sum_{x=0}^{\hat{x}}((1-x)\eta_\kappa\hat{x}) - \sum_{x=0}^{\hat{x}}\delta(x,\hat{x}) \\ - \mu_{1,1}q_{1,1} - \varphi_{1,1} - \mu_{1,2}q_{1,2} - \varphi_{1,2} - \mu_{1,3}q_{1,3} - \varphi_{1,3} \\ - \mu_{2,1}q_{2,1} - \varphi_{2,1} - \mu_{2,2}q_{2,2} - \varphi_{2,2} - \mu_{2,3}q_{2,3} - \varphi_{2,3}\end{aligned} \qquad (36.)$$

Equation 36 expresses $W$ in terms of customer numbers, wholesale transfers, and costs, plus terms that represents $\beta$, $\kappa$ and the disutility factor $\delta$. Note that the prices charged by firms to consumers do not affect social welfare $W$.

Equation 36 can be simplified further, buy reducing the wholesale transfer term. Earlier, we made two assumptions, that

$$\begin{aligned}q_{2,1} = q_{2,2} + q_{2,3} \\ \mu_{2,1} = p_{1,1^*}\end{aligned} \qquad (37.)$$

Looking only at the wholesale transfer portion of equation 36, $(p_{1,1^*} - \mu_{1,1})(q_{2,2} + q_{2,3})$, we substitute these in and get

$$\mu_{2,1}q_{2,1} - \mu_{1,1}q_{2,1} \qquad (38.)$$

The first portion of equation 38, $\mu_{2,1}q_{2,1}$, cancels out the same term from the cost components of firm 2 in equation 36, giving

$$W = \left(-\mu_{1,1}q_{2,1}\right) + \sum_{x=0}^{\hat{x}}\left(\left(1-x\right)\eta\hat{x}\right) + \sum_{x=0}^{\hat{x}}\left(\left(1-x\right)\eta_{\kappa}\hat{x}\right) - \sum_{x=0}^{\hat{x}}\delta\left(x,\hat{x}\right)$$
$$- \mu_{1,1}q_{1,1} - \varphi_{1,1} - \mu_{1,2}q_{1,2} - \varphi_{1,2} - \mu_{1,3}q_{1,3} - \varphi_{1,3} \qquad (39.)$$
$$- \varphi_{2,1} - \mu_{2,2}q_{2,2} - \varphi_{2,2} - \mu_{2,3}q_{2,3} - \varphi_{2,3}$$

which is the formula for social welfare in the case where firm 2 is purchasing unbundled elements (service 1) from it's competitor, firm 1. From equations 38 and 39, we can see that if $\mu_{1,1} < \mu_{2,1}$ social welfare will increase with the use of unbundled elements. This supports our main thesis that incumbents should be required to unbundle specific elements of their access networks if those elements meet a test for subadditivity.

THE DISUTILITY FACTOR

Finally, we look at the disutility factor $\delta\left(x,\hat{x}\right)$ for each service. This factor represents the negative cost to a consumer when they subscribe to a service that is not available from their preferred provider at a satisfactory price. We derived this for each of the games discussed above in our analysis of foreclosure opportunities.

Six games are relevant to this analysis:

- The monopoly
- Unbundled elements without tying
- Unbundled elements with tying
- The duopoly
- Exclusive enhanced services with full tying
- Exclusive enhanced services with partial tying

The second and fourth games are easy, as the logic above demonstrates that no consumer purchases service from their second choice provider, and thus the sum of their disutility factors is zero. The remaining games are representative of the issues facing regulators today in the local telecommunications access market. We will look at each in turn, and then compare the outcomes, as measured by their impact on social welfare.

We did not look at the monopoly specifically in our analysis of foreclosure opportunities, so we need to first determine the number of consumers that are taking service from the monopoly, but would prefer to get service elsewhere if it were available. Earlier, we assumed that ½ of our market were oriented to the second provider. This is $q_{2,1} = q_{2,2} + q_{2,3}$ from table 1 above. We also know that a consumer will purchase service if the offered price is below their value $\beta$ less their disutility value $\delta$. Formally, this is

$$U_x \overset{def}{=} \begin{cases} \beta_x - \delta_x - p & \text{if she subscribes} \\ 0 & \text{otherwise} \end{cases} \qquad (40.)$$

We restate this in terms of the customer's incremental impact on social welfare $\Delta W$, which is

$$\Delta W_x = \beta_x - \delta_x \qquad (41.)$$

In addition, we must account for the difference in costs between the cost of the monopoly service $\mu_1$, and the (theoretical) cost of an alternative service, $\mu_2$. As above, the price terms cancel out, leaving us with

$$\Delta W_x = \beta_x - \delta_x + \left(\mu_{2,1} - \mu_{1,1}\right) \quad (42.)$$

as the generalized impact of a single consumer on social welfare in the case of a monopoly provider. This is summed over the number of consumers that are affected, $q_{2,1}$, and stated as

$$\Delta W = \sum_{x=0}^{q_{2,1}} \beta_x - \sum_{x=0}^{q_{2,1}} \delta_x + \left(\mu_{2,1} - \mu_{1,1}\right) q_{2,1} \quad (43.)$$

We note for clarity that this represents the consumers identified as being in the range of $0 \le \hat{q} \le q_D$ in our analysis of foreclosure above. Going forward, we use the notation of $q_D$, where $0 \le q_D < q = q_{2,1}$, to remind us that the range of consumers that take services from an alternative provider value is an outcome of the game, and is not an assumption from table 1 or elsewhere. Additionally, we note that the $\beta$ factor included in equation 43 is already included in the more comprehensive definition of $W$ given in equation 39 above.

Clearly, we can see that $\Delta W$ increases if $\delta_x$ is small and/or if the monopoly's marginal costs are small relative to that of a (theoretical) competitor. It is the latter fact that supports our requirement that network elements be strongly subadditive before a regulator forces unbundling on incumbent providers of local telecommunications access services.

The first interesting game covered in our foreclosure analysis was use of unbundled elements with no tied services. The results of this game left all purchasing consumers served by the firm of their choice, which implies that the sum of all the $\delta_x$ values is zero, resulting in $\Delta W = 0$. Clearly, this maximizes $W$ with respect to $\delta_x$ in this game. For completeness, we note that the cost terms included in equation 25 above also are zero with respect to their impact on $W$, as all consumers are using the same least cost connection service via the unbundled elements.

The second interesting game involved unbundled elements tied to a service offering. This provides the most complex result, as only small portion of the consumers take the offering from their alternative provider. We restate the result of that analysis, given in equation 10, here

$$\Delta W = -\sum_{x=q_C}^{q_{ABC}} \delta_x + \left(\mu_{2,1} - \mu_{1,1}\right)\left(q_{ABC} - q_C\right). \quad (44.)$$

where $0 \le q_C < q_{ABC} < q = q_{2,1}$. In this case, the calculation of $q_C$ and $q_{ABC}$ requires knowing the exact functions for both $\beta$ and $\delta_x$. Since we have not defined a precise function for $\delta_x$, we cannot determine the exact impact here. However, another look at figure 2 will show that if the price of the competitor's service C is less than $\delta_{q_C}$, then no consumer will shift to their alternative provider, and $\Delta W = 0$. Conversely if $p_C$ is much larger than $\delta_{q_C}$, we approach the negative impact levels seen in the monopoly case.

The next game was the simple duopoly game. As in the first game, untied unbundled elements, this game left all purchasing consumers served by the firm of their choice, which implies that the sum of all the $\delta_x$ values is zero, resulting in $\Delta W = 0$. However, in this game there is the potential for different marginal cost levels which can impact *W* negatively if one provider's costs are significantly higher than the other's.

The last two games, those two concerning an exclusive enhanced service offering by firm 1, resulted in the same number of consumers taking the exclusive offer bundled with the connection service and the basic service. This included consumers in the range $0 \le \hat{q} \le q_D$ in equation 43 of our analysis of foreclosure above, and leads to

$$\Delta W = -\sum_{x=0}^{q_D} \delta_x + \left(\mu_{2,1} - \mu_{1,1}\right) q_D . \tag{45.}$$

where $0 \le q_D < q = q_{2,1}$, after accounting for the change in marginal costs. We note that this is similar to $\Delta W$ in the monopoly case from equation 43, $\beta$ term aside, and shows the power that an exclusive offer can exert on complementary competitive services.

## Comparison of Social Welfare Maximum

As we have seen, unbundling has a positive impact on social welfare, provided the marginal costs of the unbundled access facilities are below the marginal costs of any access network a competitor can construct. This would certainly be true within a single SAI if either the installation of the original access network or the competitive alternative access network exhibit the properties of subadditivity, which would imply that a single network in an SAI would be cheaper than any two or more networks serving the same neighborhood. While it is beyond the scope of this paper to prove that local access facilities are subadditive, we believe it is fair to assume that this is true[6]. Thus we assert that unbundling of local access facilities is good for social welfare, specifically because it effectively allows the elimination of the disutility factor associated with taking services from a subscribers second choice carrier.

We highlight one result of this analysis: that the firm providing the unbundled elements does not pay for this increase in social welfare, which comes from the lowering of the disutility factor, and in fact may gain from it via the subadditive effects of operating a larger access network within the SAI.

With respect to the remainder of the network behind the SAI (everything behind the SAI towards the backbone and the back office), it is not clear if a single network or a multiplicity of networks result in a higher overall level of social welfare. Since the

[6] For a more in depth discussion of subadditive costs, see Fuss and Waverman (2002), Gasmi, Laffont and Sharkey (2002), Wilson and Zhou (2001), Cave, Majumdar, and Vogelsang (2002) and Baumol and Braunstein (1977).

remainder of the network makes up the bulk of the fixed costs not attributable to any specific consumer, the outcome of such an analysis would end up weighing the balance of the disutility factors versus the fixed costs of the additional networks. Such an analysis, which is beyond the scope of this paper, would necessarily need to account for not only the network effects, but also the impacts technology and the economies of scale, scope, learning, and more. In addition, it is this authors opinion that an assumption of subadditivity is not reasonable without further analysis for any other part of the telecommunications network, especially in reference to a single SAI.

Alternative outcomes are possible in some circumstances. If the marginal cost of the installation of access facilities is sufficiently low relative to a specific consumers' or group of consumers' utility value, a second provider can install parallel facilities and make a profit. The resulting competition increases the overall social welfare.

### Recommendations

This paper proposes that *any* telecommunications carrier that installs a discrete facility that is both "essential" as described below, and is strongly "subadditive" should be given a limited-term, protected monopoly in the use of that facility, and then be required to make it available to competitors under terms similar to today's unbundling rules. The carrier uses the protected period to recover the costs of installation of the facility. Carriers should have freedom to set their own rates for the use of such facilities during this monopoly period. Once the period ends, the facility must be unbundled upon the request of a competitor, with rates for the unbundled element based on the existing total element long run incremental costing (TELRIC) mechanism since the carrier is assumed to have recovered their capital costs during the protected period.

For such a regulatory approach to work, the implementation would have to meet a number of conditions. These address the allowable rates carriers may charge for new and old services, a constraint on forcing a consumer to use the new facilities, a constraint on the facilities covered, and maintenance of the existing interconnection rules for competing carriers. The result is improvements to social welfare via increases in consumer choice, and the incentive provided to firms to install new facilities in support of innovative new services.

## Other Considerations

In this section, we briefly look at a number of issues related to the successful implementation of our suggestion to provide a protected period use for local access facilities before they are required to be unbundled. We look at the application of this approach to fiber access facilities. We quickly define what we mean by the term essential facility, and how it relates to the concept of impairment in the 1996 Act, the TRO, and this proposal. We discuss both of these issues in the next few paragraphs and we make some suggestions about how to address them to maximize the probability for the success of this proposal.

## Applicability to Fiber Loops

The FCC claimed in their TRO that because no carrier had a significant installed base of fiber in the local access plant, all had an equal opportunity to install fiber at similar costs, and thus all have a similar opportunity for a return. Thus, no carrier is an incumbent with respect to fiber access facilities under current FCC policies. We do not see how this decision aligns with the intent of Congress set forth in the Act, which expressly requires unbundling of local access loops by incumbents, subject to the 'necessary' and 'impairment' standards of section 251 (d)(2), but which does not set limits on time or technology.[7]

Leaving aside the obvious advantages of being able to move existing customers onto new facilities immediately, the FCC failed to consider the case of a carrier attempting to enter a neighborhood market some time after any previous carrier installed fiber. The first carrier to move into a neighborhood with a new technology can take advantage of subadditivity and the economic effects demonstrated above to rapidly gain a first-mover advantage over other carriers, which can result in a sustainable if localized monopoly. A provider can accelerate this result by offering some exclusive service. Subsequent entrants of any background face substantial barriers to entry, unless regulators moderate the effects of subadditivity with unbundling requirements.

In other words, the FCC and Congress have neglected to allow for the evolution of the market over time in their respective definitions of fiber facilities and ILECs. While it is perfectly sensible for Congress to specify dates in much of its legislation, the use of such a fixed point in time in the definition of a class of participants in a rapidly changing market makes less sense. A better definition of incumbent would include some definition of the extent of participation and longevity in a specified market.[8] Such a definition would allow an incumbent carrier to be classified as a new entrant with respect to some portion of its offering, in some locations, thus enabling it to gain relief from unbundling and other obligations for a limited time with respect to those offerings. We suggest that this intentional loophole will to allow a carrier an incentive to install new equipment and offer new services (separately from other services) for a limited time without the intensive obligations of incumbency and its associated unbundling currently enshrined in the Act.

## Essential Facilities

The proposed regulatory approach applies to specific "essential facilities" in the telecommunications local access market. We define essential facilities as facilities that are both necessary for the provision of an essential service[9] to a particular consumer and

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[7] 47 USC 251 (c)(3).

[8] Presumably, if someone called upon the FCC to make a ruling based on Section 251 (h)(2), it will use such a definition.

[9] These include, but are not limited to, access to local and toll calling services, operator and directory services, access to E911 and the telephone relay service for the deaf, and the capability of supporting low speed modem service for internet access. See Federal-State Joint Board on Universal Service, CC Docket No. 96-45, *Report and Order*, 12 FCC Rcd 8776 (1997) (*Universal Service Order*), as corrected by Federal-State Joint Board on Universal Service, *Errata*, CC Docket No. 96-45, FCC 97-157 (rel. June 4, 1997).

strongly subadditive. This narrow definition clearly includes access plant feeder and distribution facilities and structures, consumer drop facilities and structures, and the associated passive and active electronic and optical components that allow the access facilities to provide basic connectivity and multiplexing functions and their functional equivalents (Sharkey 2002). It does not include any functions or elements (e.g. switching) that can be easily located at an arbitrary distance from the consumer, with a shared medium providing the connection to the consumer.

Remote terminals are crucial in this regard, since they represent the first point of intelligence in the network, and thus are the first point at which both multiplexing and segregation by provider can take place (Sharkey 2002). Including basic connectivity and multiplexing functions (specifically including both packet and circuit switched capabilities as they are used in the multiplexing function) goes one step further, and captures the multi-homing functions commonly found in modern remote terminals today.

An essential facility is one that cannot be economically replaced by an alternative technology in a distant location for a specific consumer, while delivering the full suite of essential services expected by that consumer. Note that for some consumers in some locations, no specific facility need be deemed essential, as two or more alternatives may have similar capabilities at competitive cost levels, while remaining profitable for their respective owners. Such locations should be deemed fully competitive, with a corresponding lifting of most price and service obligations on all carriers serving that location or area, including any unbundling requirements as suggested in this proposal.

Economics of scope are a necessary condition for subadditivity, so let us examine them next. (Bloch, Madden and Savage (2001), but see Sung and Gort (2000) for an opposing view on the relationship between economies of scope and subadditivity.) In the local access delivery market, economies of scope come in two orthogonal aspects. First, a firm achieves economies of scope with respect to unbundleable facilities through the sharing of structures that support the delivery facility across multiple consumers. Such structures include poles, trenches, ducts, conduits and the infrastructure portions of remote terminals. Such sharing is essential to the cost-effective installation of local access facilities, even for facilities dedicated to a single consumer (e.g. a single copper loop) (Sharkey 2002).

However, note that this subadditivity only extends as far as the shared structures supporting the deliver facility. Thus, as we define the local access plant to be the outside facilities between the carriers wire center and the customer premises, such subadditivity only applies within the geographical area covered by the shared structures. This is generally the area associated with a single Service Area Interface, or SAI (Sharkey 2002). The SAI then limits the scope of price averaging under this proposal, since prices must be directly associated with the corresponding costs for this proposal to provide the proper incentives to carriers to maintain reasonable prices in the face of selective competition. Thus, for the proposal to be successful, carriers must be allowed to set charges for such facilities separately for each neighborhood or SAI.

The condition of subadditivity is narrower and simpler than the economic test for impairment that the FCC put forth in the TRO[10]. The FCC test is based first on the presence of competition in a given market, then on the potential for entry by a competitor, and was designed to maintain competitor access to switching and transport elements.[11, 12] A test based on subadditivity depends less on contestable market variables and limits the outcomes to essential facilities and structures that truly impair a competitor's ability to enter a market.

Local access facilities themselves can also exhibit economies of scope with the addition of optional or enhanced services delivered over the same facility (Berg and Tschirhart 1995). The core services are generally local basic voice services. With the addition of toll calling, call-waiting and additional services, these core facilities can show economies of scope even in the case of simple copper access facilities. The addition of packet-based services, including simple data transfer services, merely extends such economies of scope, when viewed from the perspective of a single SAI in the local access plant. However, the collection of services supplied over any single facility may or may not demonstrate economies of scope. Because this proposed definition of essential local access facilities does not include much of the network technology required to offer specific services, services should not be included in an analysis of subadditivity that marks a network facility for unbundling under this proposal. Such an analysis of service economies of scope necessarily requires an understanding of the technological elements beyond the essential local access facilities required for each service, and is beyond the scope of this paper. Happily, such an analysis is not required to support the concepts presented in this paper; only subadditivity at the facility level is required.

## CONCLUSIONS

To conclude, this paper recommends a change to the FCC's impairment standard, as set forth in their recent triennial review order, to one based largely or solely on the cost structure-based principals of subadditivity. It recommends that the FCC focus the scope of impairment so that it targets facilities and structures within the network that are strongly subadditive. Finally, the paper proposes a period of protection from the unbundling rules for new facilities that enable new services, similar to patent protection for new inventions. These changes combine to provide strong incentives to both incumbents and competitors to install new facilities and offer new services, while limiting the long term potential for the development of localized monopolies within the telecommunications industry. All of these changes enhance the overall social welfare of the area affected.

[10] *Triennial Review Order* at 84-117.
[11] See the statements issued by FCC Commissioners Powell and Abernathy at the time of the release of the TRO.
[12] Fiber access facilities are excluded because they are all new, and both incumbents and competitors face similar costs and risks when building such fiber facilities today.

## APPENDIX A: SETTING A MONOPOLY PRICE WITH NETWORK EFFECTS

This appendix is an abridged version of the material found in chapter 5 of Oz Shy's book *The Economics of Network Industries* (2001). This volume analyzes a number of networked industries, including parts of the telecommunications industry. Of particular interest is the analysis of how a monopoly firm offering a telecommunications service would set a service price that would maximize their profits. The result provides the monopoly price $p_m$ and the number of customers $\hat{x}$ served at that price. We use these values for $p_m$ and $\hat{x}$ extensively in the main body of this paper. Thus, we provide this analysis here to assist the reader in understanding the derivation of these key values.

We set the stage by looking at the case of a simple monopoly providing a service bundle over an access facility. The service bundle is assumed to have strong network effects, and can be thought of as basic local telephone service. First, borrowing from Shy (2001) we make some assumptions and derive models of a simple monopoly provider of connection services.

Following Shy's analysis (2001, p. 110ff) we begin by considering a continuum of $\eta$ potential consumers uniformly indexed by $x$ on the unit interval [0, 1] with density $\eta > 0$. We interpret consumers indexed by a low $x$ as those with a high willingness to pay, and consumers indexed with a high $x$ as those with a low willingness to pay. This provides a *cumulative density function*, which tells us for any value of $x$ how many consumers there are with index types between zero and $x$. Thus for example there are $\eta/2$ (half the population) who are types indexed on [0, ½].

We denote with $q$, $0 \le q \le 1$, the total number of consumers who subscribe to this service and with $p$ the connection price charged to subscribe to the service. We define the utility of a type $x$ consumer $0 \le x \le 1$, as

$$U_x = \begin{cases} (1-x)q^e - p & \text{if she subscribes} \\ 0 & \text{if she does not subscribe} \end{cases} \tag{1}$$

where $q^e$ is the consumers' *expected* number of customers subscribing to this service. Thus the utility of the service rises as the number of customers grows, in accordance with the concepts of network externalities.

The customers' aggregate demand for the connection service can be calculated as follows. For the customer denoted by $\hat{x}$, who is indifferent to subscribing at a given connection price $p$, (1) implies that

$$0 = (1-\hat{x})q^e - p \quad \text{or} \quad \hat{x} = \frac{q^e - p}{q^e} \tag{2}$$

This shows that consumers indexed by $x > \hat{x}$ will not subscribe to the service. Thus $q = \eta\hat{x}$ is the actual number of customers.

We make an assumption to simplify the calculations. We assume that consumers have perfect foresight, allowing them to know the number of other consumers on the network, and thus knowledge of the level of utility that a network can provide. Formally, this is

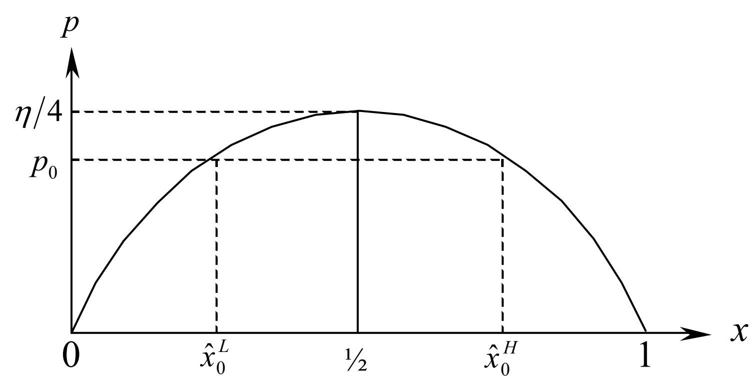
$$q^e = q = \eta\hat{x}. \tag{3}$$

Substituting (3) into (2) yields

$$p = (1-\hat{x})\eta\hat{x} \tag{4}$$

which is the inverse demand function for network connections and telecommunications services generally. This is illustrated in figure A1.

**Figure A1.**




Observe that there are two values $\hat{x}_0^H$ and $\hat{x}_0^L$ associated with a particular price $p_0$. The exact values can be determined by solving (4) for $\hat{x}_0^H$ and $\hat{x}_0^L$ to get

$$\hat{x}_0^L = \frac{\eta - \sqrt{\eta(\eta - 4p_0)}}{2\eta} \quad \text{and} \quad \hat{x}_0^H = \frac{\eta + \sqrt{\eta(\eta - 4p_0)}}{2\eta}. \tag{5}$$

These values represent two potential demand levels for a given connection price $p_0$. The lower value, $\hat{x}_0^L$, measured by $q = \eta\hat{x}_0^L$, is associated with the smallest number of consumers that have a high valuation of the service. The higher value, $\hat{x}_0^H$, measured by $q = \eta\hat{x}_0^H$, represents a higher demand level. The only stable demand equilibrium is $\hat{x}_0^H$, since a small increase in consumers at $\hat{x}_0^L$ will cause all consumers indexed in the range $[\hat{x}_0^L, \hat{x}_0^H]$ to subscribe. This gives the critical mass level for $p_0$, which is $\hat{x}_0^L$.

Assuming for the moment a simple monopoly without fixed or sunk costs, and a negligible marginal cost to add a customer, the monopoly maximizes its profits $\pi$ by solving

$$\max_{\hat{x}} \pi(\hat{x}) \stackrel{def}{=} p\eta\hat{x} = (1-\hat{x})(\eta\hat{x})^2. \tag{6}$$

This shows that the firms' profit $\pi$ is zero when $\hat{x}=0$ since there are no customers. Similarly, $\pi$ is zero when $\hat{x}=1$ since the last customer will not subscribe unless the price is zero.

The first and second order conditions for (6) are

$$0=\frac{d\pi}{d\hat{x}}=\left(2x-3x^2\right)\eta^2 \text{ and } 0>\frac{d^2\pi}{d\hat{x}^2}=(2-6x)\eta^2, \tag{7}$$

showing that $\hat{x}=0$ and $\hat{x}=2/3$ are extremes under these conditions. Because the first order condition is positive for all $0<\hat{x}<2/3$, and the second order condition is negative for $\hat{x}>1/3$, $\hat{x}=2/3$ is a global maximum. Substituting $\hat{x}=2/3$ into (4) and (6) to solve for the monopoly price $p$ and profit $\pi$ we get

$$p=(1-\hat{x})\eta\hat{x}=\frac{2\eta}{9} \text{ and } \pi=(1-\hat{x})(\eta\hat{x})^2=\frac{4\eta^2}{27}. \tag{8}$$

Note that this allows both the firm's profit and the consumers' utility level to increase with an increase in the size of the network. However, there is room for competitive entry via unbundled access to the facility under certain conditions.

As Shy (2001, p. 114) points out, restoring the connection costs in the above analysis does not change the qualitative analysis, but does change the optimal values for the monopoly provider. Given that the marginal cost of a connection is $\mu$ and the provider has a fixed cost of $\phi$, the profit maximization problem is

$$\max_{\hat{x}} \pi(\hat{x})\overset{def}{=}(p-\mu)\eta\hat{x}-\phi=[(1-\hat{x})(\eta\hat{x})-\mu]\eta\hat{x}-\phi \tag{9}$$

with first and second order conditions

$$0=\frac{d\pi}{d\hat{x}}=2\eta^2\hat{x}-3\eta^2\hat{x}^2-\eta\mu \text{ and } 0>\frac{d^2\pi}{d\hat{x}^2}=2\eta^2-6\eta^2\hat{x} \tag{10}$$

Since only $x>2/3$ fulfills the second order condition, we only need to solve for the larger root of the first order condition to get

$$\hat{x}=\frac{\eta+\sqrt{\eta(\eta-3\mu)}}{3\eta} \tag{11}$$

which converges to $\hat{x}=2/3$ as $\mu\to 0$.

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